

**33.15** 100,000gpm of water falls 900 feet through a hydroelectric turbine system with 90% efficiency. The friction loss through the system is 100 feet. How much power is generated by the system?

- A. 13MW
- B. 15MW
- C. 17MW
- D. 19MW

A hydroelectric turbine is effectively a pump acting in reverse, turning hydraulic horsepower into brake horsepower and then turning a generator, which is effectively a motor operating in reverse, turning brake horsepower into electricity. This problem gives the efficiency of the hydroelectric turbine *system*, therefore it is not necessary to analyze the detailed interaction between the turbine and the generator. Rather, the entire system can be viewed as taking water horsepower, *whp* as an input, and producing electrical power,  $P_{elec}$ , as an output.

Write the form of the **Bernoulli Equation** suited for pumping applications where state 1 is located at the source on the turbine inlet side, state 2 represents the outlet side, and  $h_A$  is the head added to the turbine (rather than by a pump).

$$h_A = \frac{P_2 - P_1}{\gamma} + \frac{v_2^2 - v_1^2}{2g} + z_2 - z_1 + h_f$$

Both the inlet and outlet states are at atmospheric pressure. There is no need to account for the 900ft of elevation differential from a static pressure perspective.

$$P_1 \approx P_2 \approx 1atm$$

$$\frac{P_2 - P_1}{\gamma} \approx 0$$

The velocity term may also be neglected as the velocities are likely to be close in magnitude at the two locations.

$$v_1 \approx v_2$$

$$\frac{v_2^2 - v_1^2}{2g} \approx 0$$

The height differential is given. Note the sign is negative due to how to states 1 and 2 were defined, i.e. state 2 is at the lower height.

$$\Delta z = z_2 - z_1 = 0ft - 900ft = -900ft$$

The friction loss is given.

$$h_f = 100ft$$

Calculate the head pressure removed by the turbine. Note this value is expected to be negative since a positive value would correspond to the head being *added*, as would occur using a pump. In the next step when calculating the water horsepower, it is perfectly acceptable to take the absolute value.

$$h_A = \frac{P_2 - P_1}{\gamma} + \frac{v_2^2 - v_1^2}{2g} + z_2 - z_1 + h_f = -900ft + 100ft = -800ft$$

The electrical power produced is a portion of the whp, as defined by the efficiency:

$$P_{elec} = \eta_T whp$$

Water horsepower is a function of volume flow rate and head:

$$whp = \frac{Q_{[gpm]} \Delta h_{[ft]}}{3960}$$

Combine these two equations.

$$P_{elec} = \frac{\eta_T Q_{[gpm]} \Delta h_{[ft]}}{3960}$$

Substitute, solve, and convert units to *KW*.

$$P_{elec} = \frac{(100,000)(800)(.9)}{3960} = 18,182hp \left( \frac{.7457KW}{1hp} \right) = 13,560KW = 13.6MW$$

**Answer A**